Simulated Annealing For Optimal IIR Digital Filters

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Abstract In this paper, Simulated Annealing (SA) in the context of designing infinite-impulse response (IIR) digital filters is presented. IIR filter is essentially a digital filter with Recursive responses. Since the error surface of digital IIR filters is generally nonlinear and multimodal, global optimization techniques are required in order to avoid local minima. This Paper presents heuristic way for the designing IIR filters. SA is a powerful global optimization algorithm introduced in combinatorial optimization problems. The paper finds the optimum Coefficients of IIR digital filter through SA. It is found that the calculated values are more optimal than fda tool availble for the design of filter in MATLAB. Design of Lowpass and High pass IIR digital filter is proposed to provide estimate of transition band. The simulation results of the employed examples shows an improvement on transition band. The position of the Pole-Zero is also presented to describe the stability of designed filters.

Keywords— Digital filters, IIR, Optimization, SA.

I. INTRODUCTION

The use of Digital filters in implementing high performance circuits in digital signal environment is becoming increasingly popular. Digital filters are capable of performance specifications that would, at best, be extremely difficult, if not impossible, to achieve with an analog implementation. In addition, the characteristics of a digital filter can be easily changed under software control. Digital filters are classified either as Finite duration impulse response (FIR) filters or Infinite duration impulse response (IIR) filters, depending on the form of impulse response of the system. In the FIR system, the impulse response sequence is of finite duration, i.e., it has a finite number of non zero terms. Digital infinite-impulse-response (IIR) filters can often provide a much better performance and less computational cost than their equivalent finite-impulse-response (FIR) filters and have become the target of growing interest. Digital IIR filters are widely used in the fields of automatic control, telecommunications, speech processing and etc. There are two main methods for IIR digital filters design. However, because the error surface of IIR filters is usually nonlinear and multimodal, conventional gradient-based design methods may easily get stuck in the local minima of error surface]. Therefore, some researchers have attempted to develop design methods based on modern heuristic optimization algorithms.

Analytical methods or Simple iterative usually leads to sub-optimal designs. Consequently, there is a need of optimization methods (heuristic type) that can be use to design digital filters that would satisfy prescribed specifications. Kirkpatrick et al. (1983) proposed an algorithm called Simulated Annealing (SA) for the solution of combinatorial optimization problems. It is inspired by the annealing process found in metallurgy

where a metal cools and then freezes to a crystalline structure. Benvenuto et al. (1992) described the salient features of using a simulated annealing (SA) algorithm in the context of designing digital filters with linear phase digital filter is presented. The algorithm is then applied to the design of FIR filter. The result was not impressive and is computationally very expensive. Oliveira et al. (2007) presented a new approach for designing linear FIR filters by using nonlinear stochastic global optimization based on simulated annealing techniques. Jung et al. (2008) found the design method of a linear phase finite word length finite-duration impulse response (FIR) filter using simulated annealing. Soyeon et al. (2011) proposed a novel pulse repetition frequency (PRF) selection scheme in medium PRF pulsed-Doppler radars using simulated annealing.

The basic limitation of all the above methods is that they can mainly be used to design FIR digital filters. The drawback of preceding design methods is that the computation time is quite long. To test the optimization procedure, the proposed algorithm is implemented in MATLAB and results are found to be very encouraging.

The outline of this paper is to introduce SA as an adaptive learning tool for the design of IIR digital filters. In this paper articles are as follows: In Section II, a brief introduction to Simulated Annealing (SA) approach is discussed. Simulated annealing related to IIR digital filter design is proposed in section III. Two designed examples are used to verify the proposed method and simulation results of experiments are reported in Section IV. The conclusion and future scope is described in end of this paper.

II. SIMULATED ANNEALING

Simulated Annealing (SA) is a well-known powerful global optimization algorithm, introduced in combinatorial optimizations .It is based on random moves, and has the ability to overcome local minima, found on the way toward a better minimum, with uphill moves. The synthesis of a digital filter transfer function requires that the values of the multiplier coefficients in the filter implementation be selected such that the transfer function of the filter meets the prescribed frequency response specification. The filter Coefficients are selected in such a way as to minimize the difference between the desired and the actual response. The design process seeks to minimize this "difference" function, known as the objective or cost function. Since the number of filter coefficients can be large, the number of configurations that the filter system can assume becomes very large. An exhaustive search for a coefficient set that minimizes the objective function can only be attempted at great computational expense. To minimize the computational effort, limited systematic or random search methods are relied upon.

Simulated annealing is a stochastic optimization technique that has been used for large-scale, discrete space problems. The simulated annealing algorithm, which is based on the theory of statistical mechanics is, essentially an iterative random search procedure with adaptive moves along the coordinate directions. It permits uphill moves under the control of a probabilistic criterion. It is the capacity to traverse "hills" that makes simulated annealing an attractive algorithm for optimizing functions with many variables and hence many local minima. A close relationship has been shown to exist between statistical mechanics and combinatorial minimization. In particular, the study shows that the process that occurs in nature in order for matter to achieve its low temperature state can provide useful techniques for use in minimization. The study also shows that finding the low temperature state alone in practice does not guarantee matter to be in one of its lowest energy states. It is possible for the substance to be in a low temperature state and yet fail to maintain equilibrium at that temperature. A system in this state is said to be "metastable", that is it can easily get out of equilibrium. Experiments that attempt to find the low temperature state of a substance begin by raising the

temperature of the substance in question to a very high value that is the melting point of the substance. The temperature is then lowered in slow stages so that at each temperature value the substance is allowed sufficient time to achieve thermal equilibrium. The process continues until the substance is cold, that is until no further changes in energy occur. The Metropolis algorithm, which has been widely used in simulated annealing, was originally developed for use in the simulation of a collection of atoms in thermal equilibrium at a given temperature. At each step of this algorithm, an atom is given a small random displacement and the resulting change in the energy of the system (ΔE) is computed. The method of simulated annealing consists of three functional relationships.

1. g(x): Probability density of state-space of D parameters x $\square\square\square$xi ; i $\square\square$1, D$\square$.
2. h(ΔE): Probability for acceptance of new cost-function given the just previous value.
3. T(k): schedule of "annealing" the "temperature" T in annealing-time steps k, i.e., of changing the volatility or fluctuations of one or both of the two previous probability densities. The new displacement is accepted subject to the criterion: If $\Delta E \leq 0$, the acceptance probability is based on the chances of obtaining a new state with "energy" Ek+1 relative to a previous state with "energy"

$$
\begin{aligned}
h(\Delta E) &= \frac{\exp(-E_{k+1}/T)}{\exp(-E/T)+\exp(-E_k/T)} \qquad (1) \\
&= \frac{1}{1+\exp(\Delta E/T)} \\
&= \exp(-\Delta E/T)
\end{aligned}
$$

where ΔE represents the "energy" difference between the present and previous values of the energies (considered here as cost functions) appropriate to the physical problem, i.e., $\Delta E = Ek\square1 \ \square\square Ek$. This essentially is the Boltzmann distribution contributing to the statistical mechanical partition function of the system. Uniform random deviates in the interval (0, l) are a convenient means of implementing the random part h (ΔE). The accepted rearrangement is then used as the starting point of the next step. By repeating the step many times, the algorithm simulates the thermal motion of atoms in thermal contact with a heat bath at temperature T. At very high values of temperature T, in the above expressions, virtually any rearrangement will be accepted. As the temperature falls to low values, rearrangements that increase the change in energy will "re likely be rejected. This strategy allows for controlled "uphill" excursions to be taken and hence to seek "hidden" arrangements with lower energy levels. Using the Metropolis algorithm, simulated annealing has been used for discrete optimization problems. Although the algorithm does not guarantee to find the global optimum, if the function has many good near optimal solutions it will find one. The simulated annealing design process consists of first "melting" the system being optimized at a high temperature and then lowering the temperature in slow stages until the system is "cold", that is until no further changes occur. As in statistical mechanics, the simulation must proceed long enough for the system to attain a steady state at each temperature. An annealing schedule is the sequence of temperatures and rearrangements of the coefficient values attempted to reach steady state at each temperature. An important feature of simulated annealing is that the "gross" features of the function appear at high values of temperature and the "finer" detail at lower values.

III. SA FOR DESIGNING IIR DIGITAL FILTERS

In general, Digital filters are classified as Recursive and Non-Recursive filters. The response of Recursive or IIR filters is dependent on one or more of its past output. IIR filters have lot of advantages over Non-Recursive filters. It has a much sharper transition characteristic for a given filter order. Like analog filters

with poles, an IIR filter usually has nonlinear phase characteristics. If such filter subjected to an impulse then its output need not necessarily become zero. The infinite impulse response of such a filter implies the ability of the filter to have an infinite impulse response. This indicates that the system is prone to feedback and instability.

Consider the IIR filter with the input-output relationship governed by:

$$y(k) + \sum_{i=1}^{M} b_i\, y(k-i) = \sum_{i=0}^{L} a_i\, x(k-i) \qquad (2)$$

where x(k) and y(k) are the filter's input and output, respectively, $M\ (\geq L)$ is the filter order. The transfer function of this IIR filter can be written as:

$$H(z) = \frac{A(z)}{B(z)} = \frac{\displaystyle\sum_{i=0}^{L} a_i\, z^{-i}}{1 + \displaystyle\sum_{i=1}^{M} b_i\, z^{-i}} \qquad (3)$$

These parameters $a_0, a_1, a_2, \ldots., a_L, b_1, b_2, \ldots.., b_M$ appearing in eq. 3 and eq. 4 are called the filter coefficients. These determine the characteristics of the filter.

The Digital filters have various stages in their design. The flow chart for the Design of Digital filter is shown in fig. 1. An important task for the designer is to find values of a_i and b_i such that the frequency response of the filter approximates a desired characteristic while preserving the stability of the designed filter.
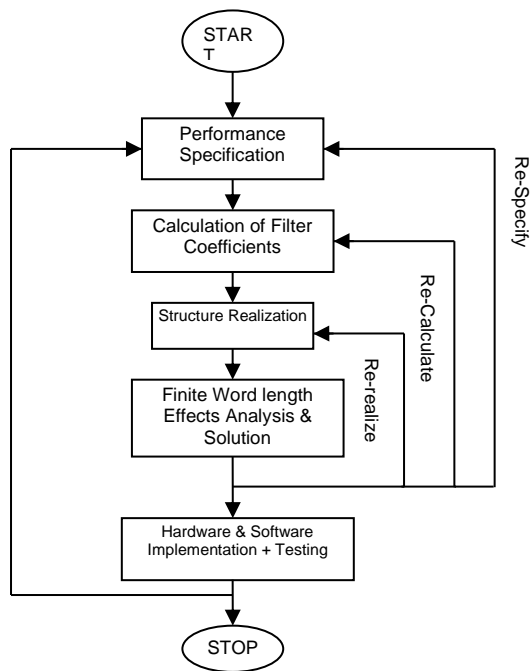

Fig. 1 Flow chart of Digital filter design

During design of digital filters, it is important to fulfill accepted design specifications such as: the width of pass-band, width of stop-band, value of pass-band ripples and value of stop-band ripples. Hence, the design of this filter can be considered as an optimization problem of cost function J(w) stated as:

min J(w)

where w=[a_0, a_1, a_2, …., a_L, b_1, b_2, ….., b_M] is filter coefficient vector. The aim is to minimize the cost function J(w) by adjusting w. The cost function is usually expressed as the time-averaged cost function defined by:

$$J(w) = \frac{1}{N} \sum_{k=1}^{N} \left(d(k) - y(k) \right)^2 \qquad (4)$$

where d(k) and y(k) are the desired and actual responses of the filter, respectively and N is the number of samples used for the calculation of cost function. The fitness value of a solution i in the population is determined by using fitness formula given as:

$$fit(i) = \frac{1}{k + J(w)i} \qquad (5)$$

where J(w) is the cost function value computed for i and k is the number of poles outside the unit circle.

The error surface of digital infinite-impulse response (IIR) filters is generally nonlinear and multimodal, so global optimization techniques are required in order to avoid local minima. In designing IIR digital filter, the values of a_i and b_i must be such that the magnitude response of the filter approximates a desired characteristic while preserving the stability of the designed filter. The summarized pseudo code of SA is shown in figure 2. In the proposed SA, various parameters fixed for analysis have been shown in table I.

```
Procedure SA
  Initialization;
  while condition not met
  new_model=random
  delta_E=E(new_model) – E(current_model)
  if (delta_E<0) current_model=new_model
   else probability P=exp^ (-delta_E /alpha_T)  :  currnt_model = new_model
  end while
end procedure
```

Fig. 2 Pseudo code of Simulated Annealing

TABLE I PARAMETERS FOR SA TECHNIQUE

Quantity	Values
Temperature Gradient	100
Initial Temperature	100
Temperature update	Exponential temperature update
Annealing Function	Fast Annealing
Function tolerance	exp^{-6}

Finishing Condition	100*5

IV. RESULTS AND DISCUSSIONS

Experiments are carried for well known IIR digital filters (see Appendix), which has been used by many authors as a "benchmark filter" for comparison purpose. To compare the performance of proposed method, the results of fda (filter design analysis) method is also obtained through simulations. The examples were performed on with Pentium IV, 2.80 GHz CPU and 1GB of RAM. Simulation study is carried out in MATLAB to demonstrate the potentiality of SA for the design of IIR digital filters. Table II provides a comparison of coefficients with the filter design tool. It is seen that the optimal coefficients provides minimum value of MSE equal to 0.3284 as compared to 0.3296 using fda. The magnitude and phase response of first example is shown in fig. 3 and fig. 4 in which a zoomed in curve for transition band is included. It is observed from the magnitude responses that the gain is -62.61 at 0.9868 using fda, but using proposed method, this gain occurs at 0.9746. This gives us better transition band compared to other technique. It is clear from phase response that phase is identical to fda tool response. In fig. 5, we have summarized Pole-Zero behavior of Low pass filter. It can be seen that the poles-zeros location of designed filter falls with in unit circle. This shows that the designed filter is also stable using proposed method.

TABLE II COEFFICIENTS OF LOW-PASS FILTER DESIGNED WITH 2ND ORDER

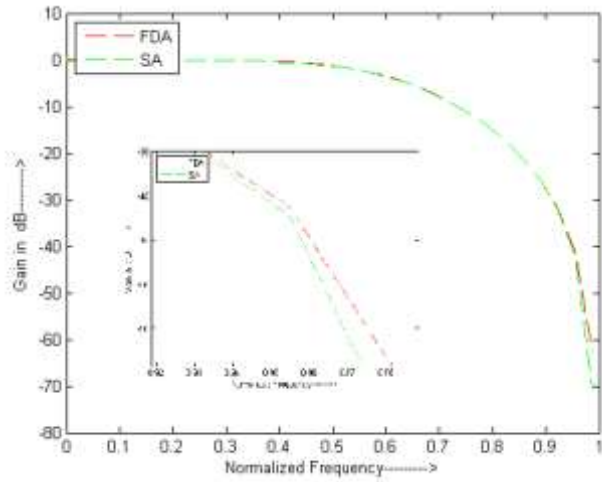| Coefficients	fda tool	SA based
a_0	0.3752	0.3749
a_1	0.7504	0.7489
a_2	0.3752	0.3749
b_1	1	1
b_2	0.3120	0.3109
b_3	0.1888	0.1790

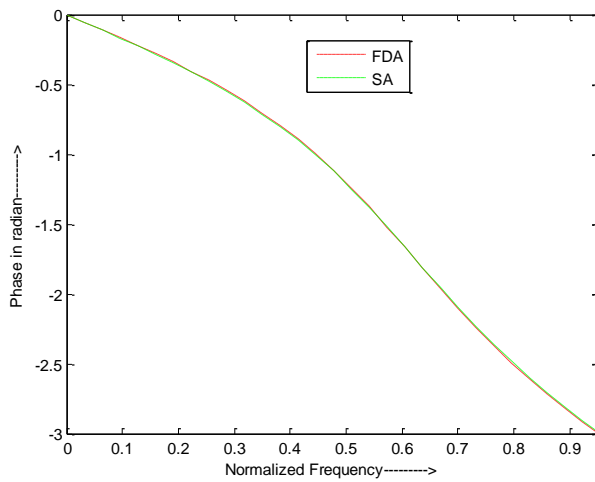Fig. 3 Magnitude Response of low pass filter

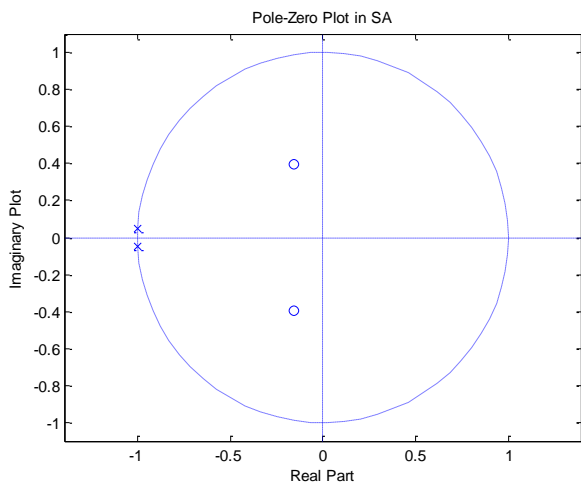Fig. 4 Phase Response of low pass filter

Fig. 5 Pole-Zero plot of low pass filter

Table III provides a comparison of coefficients of high-pass filter with the filter design tool. It is seen that the optimal coefficients provides minimum value of MSE equal to 2.9963 as compared to 3.0589 using fda. On comparison, it is found that the proposed algorithm gives optimal coefficients for High pass filter and least mean square error.

TABLE III COEFFICIENTS OF HIGH-PASS FILTER DESIGNED WITH 3RD ORDER

Coefficients	fda tool	SA based
a_0	0.0004218886	0.0004218886
a_1	-0.001687554	-0.00161024
a_2	0.0025313316	0.00229933
a_3	-0.001687554	-0.0014555
a_4	0.0004218886	0.000344561
b_1	1	1
b_2	3.177811	3.15992
b_3	3.8546544	3.80985
b_4	2.10730622	2.069939
b_5	0.43700264	0.426703

The fig. 6 and fig. 7 illustrates the magnitude and phase response of second example.
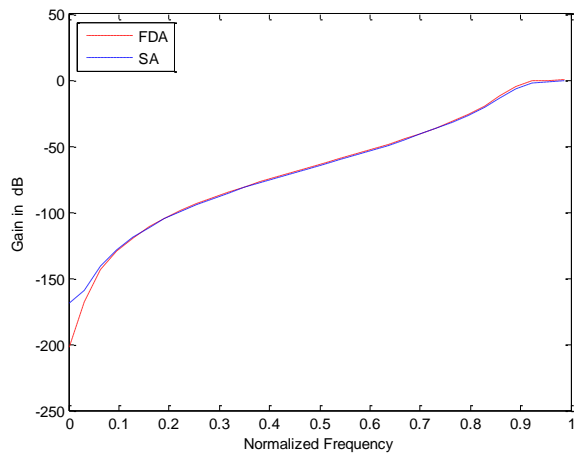

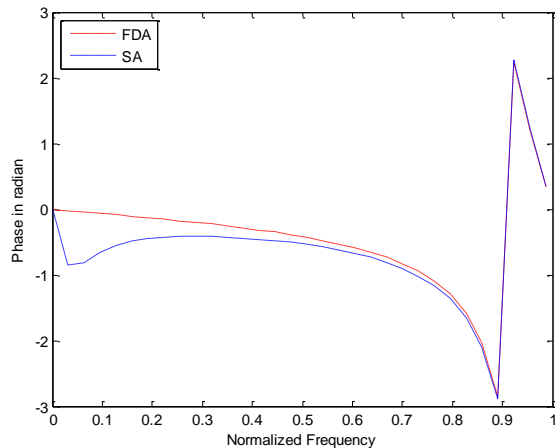Fig. 6 Magnitude Response of high pass filter
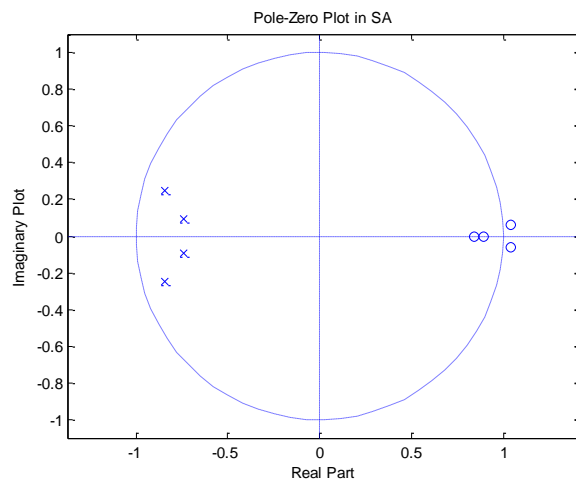
Fig. 7 Phase Response of high pass filter

Fig. 8 Pole-Zero plot of high pass filter

Fig. 8 summarized Pole-Zero position of High pass filter using SA. It is clearly seen that the poles and zeros placement gives us stable high pass filter.

V. CONCLUSION

A simulated annealing algorithm has been used for optimal IIR digital filter design. The algorithm obtains IIR filter coefficients to approximate a desired magnitude frequency response. The coefficients has been developed which are based on specifications. The values of these specifications are selected after in depth study, expert choice and literature. The Plots of Frequency Response and Pole-Zero of low pass filter and high pass filter is introduced. The Pole-Zero plots discusses the stability of IIR digital filters. The performance of proposed method was compared with fda tool. The simulation results show that SA has better, or at least equivalent, global search ability but computation time is quite long. The analysis will help the designers/ engineers to decide about the selecting coefficients to attenuate the signal. Further, the other Evolutionary algorithm can be discussed for the design of IIR filters.

APPENDIX

Example 1: In the first example, design a low-pass filter with following specifications: Pass/Stop band ripples 1dB/ 15dB, and band edges 200Hz / 400Hz and a sampling frequency of 1000Hz.

Example 2: This example is taken for design of a high-pass filter with following specifications: Pass/Stop band ripples 1dB/ 75dB, and band edges 600Hz / 200Hz and a sampling frequency of 1500Hz.

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